E–3832

M. Sc./M. A. (Final) EXAMINATION, 2021

MATHEMATICS

(Optional)

Paper Fifth (*i*)

(General Relativity and Cosmology)

Time : Three Hours]   [*Maximum Marks : 100*

Note : Attempt any *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) Let the components of velocity vector in Cartesian coordinates are $\dot{x}, \dot{y}.$ Find connesponding components in polar coordinates.

   (b) Show that $(1, 0, 0, -1)$ is a null vector whereas $(1, 0, 0, \sqrt{2})$ is a unit vector in Minkowski space.

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(c)   Show that :

$$\Gamma^i_{il} = \frac{\partial \ln \sqrt{-g}}{\partial x^l}$$

Find   also Γ^i_{il} for   the   metric

$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin\theta \, d\phi^2$.

Unit—II

2. (a)   Show that divergence of Einstein tensor is zero

i. e. $G^{\mu r}_{;r} = 0$.

(b)   Find Newtonian limit of Einstein field equations.

(c)   Show that Poisson equation is an approximation of Einstein field equations.

Unit—III

3. (a)   Find Schwarzschild interior solution when pressure p and density ρ are related as $p = -\rho$ in a perfect fluid spherically symmetric body.

(b)   Show that net deviation of the light ray due to sun is 1.74″.

(c)   Derive the Reissner-Nordström black hole solution.

Unit—IV

4. (a)   State Weyl postulate and cosmological principle. Using these derive FRW metric.

(b) Discuss Einstein-de Sitter model of the universe for dust case.

(c) Derive the formula for measuring the angular size of the galaxy. Write a short note on Hubble's law.

Unit—V

5. (a) Discuss dust model of the universe (flat case only).

(b) What is perfect cosmological principle ? Show that the principle leads to steady state universe. What do you mean by particle horizon and event horizon ?

(c) What is Olben paradox ? How does the expanding universe hypothesis resolve it ? What is gravitational red shift ?